The Gribov Conception of Quantum Chromodynamics

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Key Words QCD, Confinement, Non-perturbative Effects

Abstract A major contribution to the quest of constructing quantum dynamics of non-Abelian fieds is due to V.N. Gribov. Perturbative approach to the colour confinement, both in gluodynamics and the real world, was long considered heretic but is gaining ground. We discuss Gribov's approach to the confinement problem, centered around the rôle played by light quarks — the supercritical light quark confinement scenario. We also review some recent developments that are motivated, directly or indirectly, by his ideas.

CONTENTS

1 INTRODUCTION

One of the most challenging problems for modern theoretical physics is understanding the confinement of colour — the selection rule suggested by quantum chromodynamics (QCD) to explain the fact that quarks (and gluons) appear in the physical spectrum only *imprisoned* inside composite states — colourless mesons and baryons.

An amazing success of the relativistic theory of electron and photon fields — quantum electrodynamics (QED) — has produced a long-lasting negative impact: it taught the generations of physicists that came into the business in/after the 70's to "not to worry". Indeed, today one takes a lot of things for granted.

One rarely questions whether the alternative roads — secondary quantization, functional integral and the Feynman diagram approach — really lead to the same quantum theory of interacting fields.

One feels ashamed to doubt an elegant and powerful, but potentially deceiving, technology of translating the dynamics of quantum fields into that of statistical systems (imaginary time = Euclidean rotation trick, resulting in the *coupling* $\leftrightarrow$ *temperature* analogy).

One takes the original concept of the "Dirac sea" — the picture of the fermionic content of the vacuum — as an anachronistic model, a sort of Maxwell's mechanical ether, *I'd-rather-do-without*.

One was taught to consider the problems with field-theoretical description of point-like objects and their interactions at very small distances (ultraviolet divergences) as purely technical: *renormalize it and forget it*.

So far so good for QED, where the physical objects — electrons and photons — are direct images of the fundamental fields that one put into the local La-

grangian of the theory. In other words, *interacting* fields closely resemble their *bare* counterparts. In these circumstances, the ground state of the theory — the physical vacuum — turns out to be, in a certain sense, trivial. The rôle played by the vacuum fluctuations of electromagnetic fields and by the "Dirac sea" of negative-energy electron states, is as important as it is straightforward. Interaction with the vacuum makes the effective interaction strength $\alpha_{\mathrm{e.m.}}$ (as well as the effective electron mass) *run* with the distance. At the same time, it does not affect the *nature* of the interacting fields.

Not so clear in QCD. Here the physical hadronic states are not in one-to-one correspondence with the fundamental quarks and gluons: interaction with the QCD vacuum changes the bare fields beyond recognition. In such an environment, our preconceived ideas about QFT dynamics may turn out to be a handicap, a serious hidden obstacle on the way to attacking the confinement problem.

Vladimir Naumovich Gribov (1930–1997) belonged to a generation of physicists (now almost extinct) that *thought* about the Quantum Field Theory (QFT), that was used to *questioning* its foundations, concepts and means. An invitation to the "Gribov conception of QCD" is an invitation to *unlearn.* Learning to unlearn isn't easy. This possibly explains why the programme that Gribov formulated and was pursuing in the 90's of explaining the confinement of colour as "supercritical binding" of light quarks has yet to receive the attention it deserves from the physics community at large.

It goes without saying that Gribov could be wrong in his vision about the nature of the QCD confinement. However, it does not seem to us a good policy to be simply indifferent to what one of the creators of the modern theoretical physics had to say on the subject, to the ideas and tools he has developed during

the last half of his professional life.

This review does not attempt to cover Gribov's impact on theoretical physics. Suffices it to mention that his name is attached to many a key notion of the theory arsenal: Gribov–Froissart projection and the Gribov vacuum pole (Pomeron), Gribov factorization, Reggeon Calculus and Reggeon Field Theory (RFT), Gribov diffusion, the AGK (Abramovsky–Gribov–Kancheli) cutting rules, the Gribov bremsstrahlung theorem, Gribov–Glauber theory of relativistic multiple scattering, Gribov–Lipatov evolution equations, Gribov copies and the horizon, etc.

Let us remark that Gribov's impact on modern physics is deeper than it is generally known to be. A couple of examples will illustrate the point.

Working on the problem of the so-called strong-coupling regime of interacting Pomerons, V. Gribov and A. Migdal developed an ingenious technique for analysing dynamical systems with long-range fluctuations. Such fluctuations being typical for condensed matter physics near the critical temperature, this triggered an important breakthrough in solid state physics. The subsequent works of A. Migdal and A. Polyakov, and a contemporary more general treatment suggested by L. Kadanoff and K. Wilson, have established the scaling solution of the problem of the second order phase transitions.

In 1969, in one of his jewels *Interaction of photons and electrons with nuclei at high energies* Gribov established and described the space-time picture of particle interactions at high energies. This work found a way through the *iron curtain*.[1] Its key elements were incorporated into the famous Feynman book which laid the foundation of the parton model. The Feynman–Gribov parton model, that is.

The last 20 years of his life V. Gribov devoted to non-Abelian quantum gauge

[1] It was reported, prior to publication, by J. Bjorken at Caltech seminar.

theories, and to the QCD confinement problem in particular. His very first QCD study of 1976-77 produced a brilliant physical explanation of asymptotic freedom introduced into the physics of strong interactions by D. Gross, F. Wilczek and H. Politzer. Gribov's approach was based on an early observation of the anti-screening phenomenon made by I. Khriplovich in a pre-historic 1969, and revealed the inconsistency of the standard field-theoretical treatment of gluon fields (Gribov copies). In the late 80's Gribov suggested the quark confinement scenario based on the so-called supercritical binding of light fermions by a quasi-Coulomb interaction.

His last works remain to be discovered, understood and developed.

For V.N. Gribov, the problem of confinement as the problem of understanding the dynamics of vacuum fluctuations, and of the structure of hadrons as physical states of the theory, was always inseparable from the problem of understanding the physics of high energy hadron scattering (the Pomeron picture).

Therefore, we start by an overview of the Gribov's development of high-energy physics. The two subsequent Sections are devoted to his contribution to the quantum dynamics of non-Abelian Yang–Mills gauge fields in general, and to the basics of the Gribov light-quark confinement picture. The last Section briefly describes the *byproducts* of the QCD programme: an unpublished study of QED at short distances (a potential resolution of the notorious "Landau ghost" problem) and his picture of a composite (super-critically bound) Higgs boson.

2 HADRON INTERACTIONS AT HIGH ENERGIES

In the late fifties, when Vladimir Gribov, then a young researcher at the Ioffe Physico-Technical Institute (Leningrad, USSR) became interested in the physics

of strong hadron interactions, not much was understood about processes at high energies. The only theoretical result, derived from the first principles, was the Pomeranchuk theorem — an asymptotic equality of particle and antiparticle total scattering cross sections.

2.1 Asymptotic Behaviour $s^{\alpha(t)}$

Gribov's 1960 paper *Asymptotic behaviour of the scattering amplitude at high energies* (1) was a breakthrough. Using the so-called double-dispersion representation for the scattering amplitude, suggested by S. Mandelstam back in 1958 (2), Gribov proved an inconsistency of the then popular *black disk model* of hadron-hadron scattering. This analysis may be considered as the first building block put into the edifice of the modern theory of hadron interactions. It has demonstrated the combined power of the general principles of relativistic quantum theory — unitarity (conservation of probability), analyticity (causality) and the relativistic nature (crossing symmetry) — as applied to high-energy scattering phenomena.

By studying the analytic properties in the cross channels, he showed that the imaginary part of the scattering amplitude in the form

$$A_1(s,t) = s\, f(t) \tag{1}$$

that constituted the black disk model of diffraction in the physical region of s-channel scattering, contradicts the unitarity relation for partial waves in the crossing t-channel. To solve the puzzle, Gribov suggested the behaviour of the amplitude (for large s and finite t) in the general form

$$A_1(s,t) = s^{q(t)}\, B_t(\ln s)\,, \tag{2}$$

where B_t is a slow (non-exponential) function of $\ln s$ (decreasing fast with t) and

$q(0) = 1$ ensures the approximate constancy of the total cross section with energy.

In this first paper Gribov only analysed the constant exponent, $q(t) = 1$, having remarked on the possibility $q(t) \neq \text{const}$ as "*extremely unlikely*". Indeed, considering the t-dependence of the scattering amplitude, this would correspond to a strange picture of the radius of a hadron infinitely increasing with energy.[2] (In this particular case he proved that the cross section has to decrease at high energies, $B_t(\ln s) < 1/\ln s$, to be consistent with the t-channel unitarity.)

Soon after that Gribov became aware of the finding by T. Regge (3) that in *non-relativistic quantum mechanics*, in the unphysical region $|t| \gg s$ (momentum transfer much larger than the energy, corresponding to large *imaginary* scattering angles $|\cos\Theta| \to \infty$), the scattering amplitude has a form

$$A(s,t) \;\propto\; t^{\ell(s)}\,, \tag{3}$$

where $\ell(s)$ is the *position of the pole* of the partial wave f_ℓ in the complex plane of the orbital momentum ℓ.[3]

T. Regge found that the poles of the amplitude in the complex ℓ-plane were intimately related with bound states/resonances. It is this aspect of the Regge behaviour that initially attracted the most attention:

> "*S. Mandelstam has suggested and emphasized repeatedly since 1960 that the Regge behaviour would permit a simple description of dynamical states (private discussions). Similar remarks have been made by R. Blankenbecker and M.L. Goldberger and by K. Wilson.*" (4)

[2] Reportedly, it was L.D. Landau who "forced" Gribov to publish Eq. (2) in its general form, with $q(t) \neq \text{const}$. "*You are too young to judge*" the blessing went, according to legend.

[3] Gribov apparently learned about the Regge result from a paper by G. Chew and S. Frautschi of 1960 which contained a *footnote* describing the main Regge findings.

The structure of the Regge amplitude Eq. (3) motivated Gribov to return to the consideration of the case of t-dependent exponent in his general high-energy ansatz Eq. (2) that was dictated by t-channel unitarity. His letter to ZhETF *Possible asymptotic behaviour of elastic scattering* (5) became the first application of Regge ideas to the high-energy asymptotic behaviour of scattering amplitudes.

By then M. Froissart had already proved his famous theorem that limits the asymptotic behaviour of the total cross sections (6),

$$\sigma^{\mathrm{tot}} \propto s^{-1}\,|A_1(s,0)| \;<\; C\,\ln^2 s\,. \tag{4}$$

Thus, having accepted $\ell(0)=1$ for the rightmost pole in the ℓ-plane — the *vacuum pole* — as the condition "*that the strongest possible interaction is realized*", Gribov formulated and discussed "*the main properties of such an asymptotic scattering behaviour ... which in spite of having a few unusual features is theoretically feasible and does not contradict the experimental data*":

- the total interaction cross section is constant at high energies,
- the elastic cross section tends to zero as $1/\ln s$,
- the scattering amplitude is essentially imaginary,
- the significant region of momentum transfer in elastic scattering shrinks with energy increasing, $\sqrt{-t} \propto (\ln s)^{-1/2}$.

He also analysed the s-channel partial waves to show that for small impact parameters $\rho < R$ their amplitudes fall as $1/\ln s$, while the interaction radius R increases with energy as $\rho \propto \sqrt{\ln s}$. He concluded:

> "*this behaviour means that the particles become grey with respect to high-energy interaction, but increase in size, so that the total cross section remains constant.*"

Thus, *shrinkage of the diffractive peak* was predicted, which was then experimentally verified at particle accelerator experiments in Russia (IHEP, Serpukhov), Switzerland (CERN) and the US (FNAL, Chicago), as were the general relations between the cross sections of different processes, that followed from the *Gribov factorization theorem* (7).

These were the key features of what has become known, quite ironically, as the "Regge theory" of strong interactions at high energies.

On the opposite side of the *curtain*, the basic properties of the Regge pole picture of forward/backward scattering were formulated half a year later by G. Chew and S. Frautschi (8). In particular, they suggested "*the possibility that the recently discovered ρ meson is associated with a Regge pole whose internal quantum numbers are those of an $I = 1$ two-pion configuration*," and conjectured the universal high-energy behaviour of backward $\pi^+\pi^0$, K^+K^0 and pn scattering due to ρ-reggeon exchange. Chew and Frautschi also stressed that the hypothetical Regge pole with $\alpha(0) = 1$ responsible for forward scattering possesses quantum numbers of the *vacuum*.

Dominance of the Gribov vacuum pole automatically validates the Pomeranchuk theorem. The name "Pomeron" for the vacuum pole was coined by Murray Gell-Mann, who referred to Geoffrey Chew as an inventor.

2.2 Complex Angular Momenta in Relativistic Theory

In non-relativistic quantum mechanics the interaction Hamiltonian allows for scattering partial waves to be considered as analytic functions of complex angular momentum ℓ. Gribov's paper *Partial waves with complex orbital angular momenta and the asymptotic behaviour of the scattering amplitude* showed that

the partial waves with complex angular momenta can be introduced in a relativistic theory as well. Here it is the *unitarity* in the crossing channel that *replaces the Hamiltonian* and leads to analyticity of the partial waves in ℓ. The corresponding construction is known as the "Gribov–Froissart projection" (9).

Few months later Gribov demonstrated that the simplest (two-particle) t-channel unitarity condition indeed generates *moving poles* in the complex ℓ-plane. This was the *proof* of the Regge hypothesis in relativistic theory (10).

The "Regge trajectories" $\alpha(t)$ combine hadrons into families: $s_h = \alpha(m_h^2)$, where s_h and m_h are the spin and the mass of a hadron (hadronic resonance) with given quantum numbers (baryon number, isotopic spin, strangeness, etc.) (8). Moreover, at negative values of t, that is in the physical region of the s-channel, the very same function $\alpha(t)$ determines the scattering amplitude, according to Eq. (2). It looks *as if* high-energy scattering was due to t-channel exchange of a "particle" with spin $\alpha(t)$ that varies with momentum transfer t — the "reggeon".

Thus, the high-energy behaviour of the scattering process $a + b \to c + d$ is linked with the spectrum of excitations (resonances) of low-energy scattering in the dual channel, $a+\bar{c} \to \bar{b}+d$. This intriguing relation triggered many new ideas (bootstrap, the concept of duality). Backed by the mysterious *linearity* of Regge trajectories relating spins and squared masses of observed hadrons, the duality ideas, via the famous Veneziano amplitude, gave rise to the concept of hadronic strings and to development of string theories in general.

2.3 Interacting Pomerons

A lot of theoretical effort was invested into understanding of the approximately constant behaviour of total cross sections at high energies.

To construct a full theory that would include the Pomeron trajectory with the maximal "intercept" $\alpha_P(0)=1$ that respects the Froissart bound, and would be consistent with unitarity and analyticity proved to be very difficult. This is because multi-Pomeron exchanges become essential, which generate *branch points* in the complex ℓ-plane, which singularities were first discovered by Mandelstam in his seminal paper (11). Moreover, the study of particle production processes with large rapidity gaps led Gribov, Pomeranchuk and Ter-Martirosyan to the concept of *interacting reggeons*. By the end of the 60-s V. Gribov had proposed the general theory known as Gribov Reggeon Calculus. He formulated the rules for constructing the field theory of interacting Pomerons — the Reggeon Field Theory (RFT) — and developed the corresponding diagram technique. Gribov RFT reduces the problem of high energy scattering to a non-relativistic QFT of interacting particles in 2+1 dimensions.

2.4 Gribov Partons and Feynman Partons

One of Gribov's most important contributions to high energy hadron physics was the understanding of the space-time evolution of high energy hadron-hadron and lepton-hadron processes, in particular the nature of the reggeon exchange from the s-channel point of view. In 1973, in his lecture at the LNPI Winter School (12), Gribov outlined the general phenomena and typical features that were characteristic for high energy processes in any QFT.

To understand the structure of hard (deep inelastic) photon–hadron interactions Feynman suggested the idea of partons — point-like constituents of hadronic matter. Feynman defined partons in the infinite momentum frame to suppress vacuum fluctuations whose presence would have undermined the notion of the

parton wave function of a hadron (13).

The power of Gribov's approach lied in applying the universal picture of fluctuating hadrons to both *soft* and *hard* interactions. Gribov's partons are constituents of hadron matter, components of long-living fluctuations of the hadron projectile, which are responsible for soft hadron-hadron interactions: total cross sections, diffraction, multiparticle production, etc.

Gribov's earlier work *Interaction of γ-quanta and electrons with nuclei at high energies* (14) had been a precursor to the famous Feynman paper. Gribov described the photon interaction in the rest frame of the target nucleus. An incident real photon (or a virtual photon in the deep inelastic scattering case) fluctuates into hadrons before the target, at the longitudinal distance L increasing with energy.[4] Therefore, at sufficiently large energy, when the fluctuation distance exceeds the size of the target, the photon no longer behaves as a point-like weekly interacting particle. Its interaction resembles that of a hadron and becomes "black", corresponding to complete absorption on a large nucleus.

Being formally equivalent to Feynman's treatment, Gribov's approach is better suited for the analysis of deep inelastic phenomena at very small Bjorken x, where the interaction becomes actually strong, and the perturbative QCD treatment is bound to fail. Gribov diffusion in the impact parameter space giving rise to energy increase of the interaction radius and to the reggeon exchange amplitude, coexisting fluctuations as a source of branch cuts, duality between hadrons and partons, a common basis for hard and soft elastic, diffractive and inelastic process — these are some of the key features of high energy phenomena in QFTs, which

[4]For the $e^- p$ deep inelastic scattering B.L. Ioffe has shown that the assumption of Bjorken scaling implies $L \sim 1/2xm_N$, with x the Bjorken variable and m_N the nucleon mass (15).

are still too hard a nut for QCD to crack.

2.5 Gribov Reggeon Field Theory

The two best known applications of the Gribov RFT are

- general quantitative relation between the shadowing phenomenon in hadron-hadron scattering, the cross section of diffractive processes and inelastic multi-particle production, known as "Abramovsky-Gribov-Kanchelli cutting rules" (AGK) (16), and
- the essential revision by Gribov of the Glauber theory of nuclear shadowing in hadron-nucleus interactions (17).

In 1968 V.N. Gribov and A.A. Migdal demonstrated that the scaling behaviour of the Green functions emerged in QFT in the strong coupling regime (18). As we have mentioned in the Introduction, their technique helped to build the quantitative theory of second order phase transitions and to analyse critical indices characterising the long-range fluctuations near the critical point.

The problem of high energy behaviour of soft interactions remained unsolved, although some viable options were suggested. In particular, in *Properties of Pomeranchuk Poles, diffraction scattering and asymptotic equality of total cross sections* (19) Gribov has shown that a possible consistent solution of the RFT in the so-called *weak coupling* regime calls for the formal asymptotic equality of *all* total cross sections of strongly interacting particles.

Gribov's last work in this subject was devoted to the intermediate energy range and dealt with interacting hadron fluctuations (20).

The study of the *strong coupling* regime of interacting reggeons (pioneered by A.B. Kaidalov and K.A. Ter-Martirosyan) led to the introduction of the *bare*

Pomeron with $\alpha_P(0)>1$. The RFT based on t-channel unitarity should enforce the s-channel unitarity as well. The combination of increasing interaction radius and the amplitudes in the impact parameter space which did not fall as $1/\ln s$ (as in the one-Pomeron picture) led to logarithmically increasing asymptotic cross sections, resembling the Froissart regime (and respecting the Froissart bound Eq. (4)). The popularity of the notion of the bare Pomeron with $\alpha_P(0)>1$ is based on experiment (increasing $\sigma_{\rm tot}$). Psychologically, it is also supported by the perturbative QCD finding that the (small) scattering cross section of two small-transverse-size objects should increase with energy in a power-like fashion in the restricted energy range — the famous "hard BFKL Pomeron" (21).

2.6 Reggeization and Pomeron Singularity in Gauge Theories

In the mid-sixties Gribov initiated the study of double logarithmic asymptotics of various processes in QED, making use of the powerful technique he had developed for the analysis of the asymptotic behaviour of Feynman diagrams (22).

In particular, in 1975 Gribov, Lipatov and Frolov studied the high energy behaviour of QED processes from the point of view of "Regge theory". High energy scattering amplitudes with exchange of an *electron* in the t-channel acquire, in higher orders in QED coupling, a characteristic behaviour $A\propto s^{j(t)}$ with $j(m_e^2)=1/2$. This means that electron becomes a part of the Regge trajectory: *reggeizes*.[5] For the *vacuum channel*, however, they found (23) that the rightmost singularity in the complex j-plane is *not a moving pole* (as it is for electron) but, instead, *a fixed branch point* singularity positioned *to the right* from unity, $j(0)=1+c\alpha^2>1$. This was a precursor of a similar result found later by Fadin,

[5] And so do quarks and gluons in QCD; Fadin, Frankfurt, Lipatov, Sherman (1976).

Lipatov and Kuraev (21) in non-Abelian theories, and QCD in particular. The problem of apparent anti-Froissart behaviour of the perturbative "hard Pomeron" in QCD still awaits resolution.

With the advent of QCD as a microscopic theory of hadrons and their interactions, the focus of theoretical studies has temporarily shifted away from Gribov–Regge problematics to "hard" small-distance phenomena.

3 NON-ABELIAN GAUGE THEORIES

V.N. Gribov became interested in non-Abelian field theories in 1976. His very first study, as a QCD apprentice, produced amazing results. In February 1977 in the proceedings of the 12^{th} Leningrad Winter School he published two lectures which were to change forever the non-Abelian landscape (24, 25).

3.1 Anatomy of Asymptotic Freedom

In the first lecture *Instability of non-Abelian gauge fields and impossibility of the choice of the Coulomb gauge* (24) Gribov gave an elegant physically transparent explanation of the *asymptotic freedom* introduced by D. Gross & F. Wilczek and H.D. Politzer in 1973 (26).

The *anti-screening* phenomenon had been first observed back in 1969 (27) for the non-Abelian $SU(2)$ theory in the ghost-free Coulomb gauge within the Hamiltonian approach. In the Hamiltonian language, there are (or rather seem to be) N^2–1 massless $\mathbf{B}$ quanta (transverse gluons, $\perp$) and, as in QED, an additional Coulomb field ($\mathbf{0}$). It is important to stress that the latter is not a physical quantum degree of freedom but a means for describing classical instantaneous interaction between colour charges. Unlike QED, the non-Abelian Coulomb field

has a colour charge of its own. Therefore, traversing the space between two external charge, it may virtually decay into two transverse fields,

$$\mathbf{0} \to \perp + \perp \to \mathbf{0}\,, \tag{5a}$$

or into a $q\bar{q}$ pair

$$\mathbf{0} \to \mathbf{q} + \bar{\mathbf{q}} \to \mathbf{0}\,, \tag{5b}$$

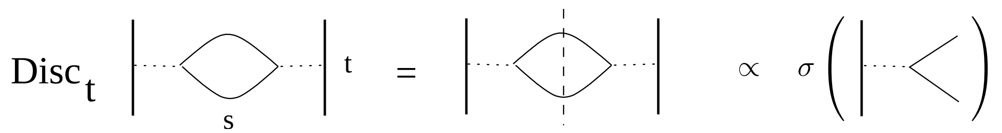
in the same manner as the QED Coulomb field fluctuates in the vacuum into an e^+e^- pair. Both these effects lead to *screening* of the colour charge of the external sources, in a perfect accord with one's physical intuition.

This (*anti-asymptotically-free*) behaviour of the running coupling was first found by Landau, Abrikosov and Khalatnikov for QED (28). It turned out to be common for all then-known renormalizable field theories: with scalar ($\lambda\phi^4$), Yukawa, four-fermion interactions (29) as well as for pedagogically valuable Lee model (30).

This observation had dramatic consequences: the physical interaction (renormalized coupling) was predicted to *vanish* in the limit of a point-like bare interaction, $\Lambda_{\rm UV} \to \infty$. In the late 1950s the problem was known as "Moscow Zero". The depth of the crisis can be measured by the Dyson prophecy (31) that the correct "meson" theory – the theory of strong interactions – "*will not be found in the next hundred years*" and by the Landau conclusion (32) that "*the Hamiltonian method for strong interactions is dead and must be buried, although of course with deserved honour.*"

Universality of the screening phenomenon was readily understood as a consequence of *unitarity* in the cross-channel. Indeed, the discontinuity of the loop

diagram at $t > 0$,



describes, according to Cutkosky rules, production cross section of two physical (*sic*!) particles in the t-channel. Positivity of the imaginary part at $t > 0$ then dictates the *screening* sign of the virtual loop in the s-channel ($s > 0$, $t < 0$).

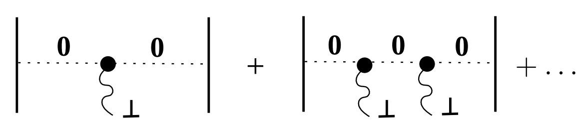
Thus, the fact that there are "physical" fields in the intermediate state in Eq. (5) — quark-antiquark or two transverse gluons — fixes the sign of the virtual correction to correspond to *screening*, via the unitarity relation in the cross-channel. Technically speaking, these virtual decay processes contribute to the QCD β-function as

$$\left\{ \frac{d\,\alpha_s^{-1}(R)}{d\ln R} \right\}^{\rm phys} \propto \frac{1}{3}N + \frac{2}{3}n_f \,, \tag{6a}$$

that is, make the effective coupling *decrease* at large distances R between the external charges, as in QED.

Where then the *anti-screening* comes from? It originates from another, specifically non-Abelian, effect namely, interaction of the Coulomb field with the field of "zero-fluctuations" of transverse gluons in the vacuum,

$$\sum_n \Big[\mathbf{0} + \perp \to \mathbf{0}\Big]^n \quad = $$



In a course of such multiple rescattering, the Coulomb "quantum" preserves its identity as an *instantaneous* interaction mediator, and therefore is not affected by the unitarity constraints. For $n = 1$ the contribution vanishes upon averaging. Statistical average over the transverse vacuum fields in the second order of perturbation theory ($n = 2$) results in an additional contribution to the Coulomb

interaction energy which, translated into the running coupling language, gives

$$\left\{\frac{d\,\alpha_s^{-1}(R)}{d\ln R}\right\}^{\mathrm{stat}} \;\propto\; -4\,N\,. \tag{6b}$$

According to Gribov, an anti-intuitive minus sign in Eq. (6b) has its own simple explanation. It is of the same origin as the minus sign in the shift of the energy of the ground state of a quantum-mechanical system under the second order in perturbation:

$$\delta E \equiv E - E_0 = \sum_n \frac{|\langle 0|\delta V|n\rangle|^2}{E_0 - E_n} \;<\; 0\,.$$

The rôle of perturbation δV is played by the vacuum field of transverse gluons.

Taken together, the two contributions Eq. (6) combine into the standard QCD β-function:

$$\frac{d}{d\ln Q^2}\frac{\alpha_s(Q^2)}{4\pi} = -b_0\left(\frac{\alpha_s(Q^2)}{4\pi}\right)^2 + \mathcal{O}\left(\alpha_s^3\right), \qquad b_0 = \frac{11N_c}{3} - \frac{2N_f}{3}.$$

3.2 Infrared Instability

The three-dimensional transversality condition

$$(\boldsymbol{\nabla}\cdot\mathbf{A}) \equiv \frac{\partial A_i}{\partial x_i} = 0\,, \qquad i = 1,2,3 \tag{7}$$

is usually imposed on the field potential to describe massless vector particles (the Coulomb gauge). Being antisymmetric, the field strength tensor $F^a_{\mu\nu}$ does not contain *time derivative* of the zero-component of the potential A^a_0. In the Hamiltonian language, this puts A^a_0 in a position of a *cyclic variable* which does not constitute a physical degree of freedom. It can be eliminated from field dynamics (24, 33), contributing to the Hamiltonian that describes transverse gluons,

$$H_{\mathrm{glue}} = \tfrac{1}{2}\int d^3x\,\left(\mathcal{J}^{-\frac{1}{2}}\,\mathbf{E}^a_\perp\,\mathcal{J}\cdot\mathbf{E}^a_\perp\mathcal{J}^{-\frac{1}{2}} \;+\; \mathbf{B}^a_\perp\cdot\mathbf{B}^a_\perp\right), \tag{8a}$$

an additional term responsible for *Coulomb interaction* between "charges",

$$H_{\rm Coul} = \frac{1}{2} \int d^3x\, d^3y\; \mathcal{J}^{-\frac{1}{2}} \rho^a(x)\, \mathcal{J}\, K^{ab}(x,y;\mathbf{A}_\perp)\, \rho^b(y)\, \mathcal{J}^{-\frac{1}{2}}, \tag{8b}$$

where ρ^a is given by the sum of the colour charge density of external sources (e.g., static quarks) and that induced by the transverse gluons themselves,

$$\rho \;=\; \rho_q \;+\; ig_s\,[\mathbf{A}_\perp, \mathbf{E}_\perp]. \tag{8c}$$

Introducing the covariant derivative $\mathbf{D}$,

$$\mathbf{D}\,[\mathbf{A}]\,C \;=\; \boldsymbol{\nabla}\,C + ig_s\,[\mathbf{A}, C]\,,$$

the Coulomb energy "propagator" K in Eq. (8b) reads

$$K^{ab}(x,y;\mathbf{A}) \;=\; -\left[\frac{1}{\mathbf{D}[\mathbf{A}]\cdot\boldsymbol{\nabla}}\,\boldsymbol{\nabla}^2\,\frac{1}{\mathbf{D}[\mathbf{A}]\cdot\boldsymbol{\nabla}}\right]^{ab}_{xy}. \tag{9}$$

We see that the propagation of a Coulomb field A_0 in the "external" transverse gluon field $\mathbf{A}_\perp$ is governed by the operator that resembles the propagator of the Faddeev–Popov ghost,

$$(\mathbf{D}\cdot\boldsymbol{\nabla})\,A_0 \;\equiv\; \boldsymbol{\nabla}^2 A_0 + ig_s\,[\mathbf{A}_\perp, \boldsymbol{\nabla} A_0]\,. \tag{10}$$

The factor $\mathcal{J}$ in Eq. (8) is the determinant of this operator,

$$\mathcal{J} = \mathcal{J}[\mathbf{A}] \;=\; -\det\left(\mathbf{D}[\mathbf{A}]\cdot\boldsymbol{\nabla}\right).$$

Taking the expectation value of $K(x,y;\mathbf{A})$ over the vacuum fields $\mathbf{A}_\perp$ produces an *instantaneous* interaction term,

$$\begin{aligned}
\left\langle A_0^a(x) A_0^b(y)\right\rangle \;&=\; G(\mathbf{x}-\mathbf{y})\delta^{ab}\delta(x_0-y_0) \\
&+\;\; \text{non-instantaneous}, \\
G(\mathbf{x}-\mathbf{y})\,\delta^{ab} \;&=\; -\left\langle\left[\frac{1}{\mathbf{D}[\mathbf{A}_\perp]\cdot\boldsymbol{\nabla}}\,\boldsymbol{\nabla}^2\,\frac{1}{\mathbf{D}[\mathbf{A}_\perp]\cdot\boldsymbol{\nabla}}\right]^{a,b}_{x,y}\right\rangle.
\end{aligned} \tag{11}$$

This expression describes an interaction generated by exchange of a Coulomb gluon *dressed* by the fluctuations of transverse gluon fields in the vacuum.

By setting $\mathbf{A}_\perp = 0$ we would return to the Laplace operator $G = -1/\boldsymbol{\nabla}^2 \propto 1/\mathbf{k}^2$ that corresponds, in the coordinate space, to the canonical Coulomb potential $1/|\mathbf{x}-\mathbf{y}|$. For small vacuum fields, $g_s A_\perp/\nabla \ll 1$, expanding perturbatively the Coulomb propagator G in Eq. (11) to the second order in g_s produces the one-loop anti-screening effect as stated in Eq. (6b).

If we take, however, gluon fields in the QCD vacuum as large as

$$g_s A_\perp/\nabla \sim g_s\, A_\perp \cdot L \sim 1$$

(with L a spatial extent of the field), the perturbative expansion in $g_s\mathbf{A}_\perp$ of the denominators in Eq. (11) is no longer justified. Under such circumstances a qualitatively new phenomenon takes place namely, the Coulomb (ghost) propagator may become singular:

$$(\mathbf{D}[\mathbf{A}_\perp]\cdot\boldsymbol{\nabla})\; C_0 \;=\; \boldsymbol{\nabla}^2 C_0 + i g_s\,[\,\mathbf{A}_\perp, \boldsymbol{\nabla} C_0\,] \;=\; 0\,. \tag{12}$$

Appearance of a “zero-mode” solution C_0 in the external field is a sign of an infrared instability of the theory.

An illuminating way to see the instability of the perturbative vacuum, and to re-derive the running of the coupling in QCD, is to look at the quantum correction to the vacuum energy $V_0(H) = \frac{1}{2}H^2$ in a constant chromo-magnetic field H.

The one-loop corrected energy density reads (34)

$$\mathrm{Re}\, V(H) = \frac{1}{2}H^2 + (g_s H)^2 \frac{b}{32\pi^2}\left(\ln\frac{g_s H}{\mu^2} - \frac{1}{2}\right) \;\simeq\; \frac{g_s^2(\mu^2)}{g_s^2(H)}\cdot V_0(H), \tag{13}$$

where μ is the renormalization scale of the bare coupling, $g_s = g_s(\mu^2)$, and

$b = 11N_c/3$ (gluodynamics). For relatively small fields,

$$H \ < \ H_0 \simeq \frac{\mu^2}{g_s(\mu^2)} \exp\left(-\frac{16\pi^2}{bg^2(\mu^2)}\right),$$

the potential Eq. (13) formally turns *negative* and develops a minimum — "true vacuum" (?) — corresponding to non-zero expectation value of the chromomagnetic field. It was soon realized, however, that this new "vacuum" is unstable (35).

In pure gluodynamics the vacuum correction can be computed by summing over the Landau levels of gluons moving in the external field:

$$\delta V(H) = \frac{g_s H}{4\pi^2} \int dp_z \sum_{n=0}^{\infty} \sum_{s_z=\pm 1} \sqrt{2g_s H(n + \tfrac{1}{2} - s_z) + p_z^2}. \tag{14}$$

The real part of (14) yields (13). Thus, the asymptotic freedom in this approach can be seen as arising from the paramagnetic response of the vacuum.

At the same time, we readily see that one specific state in Eq. (14) namely, $n = 0$ with gluon spin parallel to the direction of the field, $s_z = 1$, gives an *imaginary* contribution:

$$\mathrm{Im}\, \delta V(H) = \frac{g_s H}{4\pi^2} \int_{-g_s H}^{g_s H} dp_z \, \mathrm{Im} \sqrt{p_z^2 - g_s H - i0} = -\frac{g_s^2 H^2}{8\pi} = -\frac{g_s^2}{4\pi} \cdot V_0(H). \tag{15}$$

Non-zero imaginary part of the effective potential signals that the vacuum is unstable (decays with time) and does not correspond to the true ground state of the theory. Note also that $n = 0$ corresponds to the Landau level of the largest transverse radius, i.e. to the *infrared* region, as had to be expected.

Once again, we come to the conclusion that naïve perturbative treatment of non-Abelian gauge fields is flawed due to problems in the infrared.

3.3 Gribov Copies

In (24) Gribov localized the problem. He showed that the three-dimensional transversality condition Eq. (7) actually *does not* solve the problem of gauge fixing.

Consider a group Ω, and let $\omega(x)$ be a function with values in this group. The basic principle that defines the corresponding gauge theory is that the vector potentials $A_\mu(x)$ and

$$A_\mu^{[\omega]}(x) = \omega(x) A_\mu(x) \omega^{-1}(x) + \partial_\mu \omega(x) \omega^{-1}(x) \tag{16}$$

describe physically identical fields. Therefore, to avoid multiple counting, one has to impose a gauge-fixing condition of the general form

$$F(A^{[\omega]}; x) = 0. \tag{17}$$

To do the job, Eq. (17) should have a unique solution for arbitrary A. This can be achieved for a variety of gauge fixings within perturbation theory, when the fields are *weak*. In general, however, solutions may appears copious since the gauge fixing condition Eq. (17) constitutes a system of nonlinear equations for the function $\omega(x)$.

Indeed, Gribov found that due to essential non-linearity of the gauge transformation Eq. (16), a "transverse" field potential satisfying Eq. (7) may actually happen to be a *pure gauge field* which should not be separately counted as an additional physical degree of freedom. He explicitly constructed such "transverse gauge fields" for the $SU(2)$ gauge group and showed that the *uncertainty* in gauge fixing arises when the effective magnitude of the field becomes large,

$$A_\perp \cdot L \sim \frac{1}{g_s},$$

or, in other words, when the effective interaction strength (QCD coupling) becomes of the order of unity, that is, in the non-perturbative region. More precisely, he found that it happens exactly when the Faddeev–Popov operator acquires a zero mode solution Eq. (12) that is, as we discussed above, in the infrared region where the vacuum enhancement of the dressed Coulomb gluon propagator (11) becomes catastrophically large.

Thus, the "surface" $(\mathbf{D}[\mathbf{A}_\perp] \cdot \boldsymbol{\nabla})C_0 = 0$ in the functional $\mathbf{A}_\perp$–space marks the border (the "Gribov horizon") beyond which the solution of the gauge-fixing equation Eq. (7) becomes copious. From this point of view, the fact that the Coulomb propagator develops singularity does not necessarily mean that the Faddeev–Popov ghost "rises from the dead" by pretending to propagate as a particle. It rather tells us that we have failed to formulate the quantum theory of interacting non-Abelian vector fields, to properly fix physical degrees of freedom.

The existence of "Gribov copies" means that the standard Faddeev–Popov prescription for quantizing non-Abelian gauge theories is, strictly speaking, incomplete and has to be modified.

A number of studies explored further the emergence of Gribov copies. It was found, both analytically (36) and numerically (37) that Gribov copies can exist even *inside* the Gribov horizon, and that a more narrow "fundamental modular region" had to be defined to avoid the problem. Nevertheless, recently it has been shown (38) that the copies within the Gribov horizon actually do not contribute to any expectation values, and thus for all practical purposes the original Gribov's recipe of constraining Faddeev-Popov determinant to positive values is correct. A promising attempt to implement the Gribov fundamental domain in the 5-dimensional formulation of gluodynamics can be found in (39).

3.4 Coulomb Confinement

Gribov himself addressed the quest of possible modification of the QCD quantization procedure in the second lecture "Quantization of non-Abelian gauge theories" (25). The paper (40) under the same title based on the two Winter School lectures is now a universally accepted (though disturbing) truth and during 25 odd years since its appearance in 1978 was being cited more than 660 times, with increasing frequency.

To properly formulate non-Abelian field dynamics, Gribov suggested to limit the integration over the fields in the functional integral to the so-called *fundamental domain*, where the Faddeev–Popov determinant is strictly positive (the region in the functional space of transverse fields $\mathbf{A}_\perp$ *before* the first zero mode Eq. (12) emerges).

Gribov produced qualitative arguments in favour of the characteristic modification of the gluon propagator, due to the new restriction imposed on the functional integral. Effective suppression of large gluon field results, semi-quantitatively, in an infrared singular polarization operator $\Pi \propto k^{-2}$,

$$D^{-1}(k) = k^2 + \Pi(k^2) \;\simeq\; k^2 + \frac{\sigma^2}{k^2}. \tag{18a}$$

The gluon Green function coincides with the perturbative one at large momenta (small distances), $D(k) \propto k^{-2}$ but *vanishes* off at $k = 0$, instead of having a pole corresponding to massless gluons:

$$D(k) \propto \frac{k^2}{k^4 + \sigma^2}. \tag{18b}$$

The new non-perturbative parameter σ^2 in Eq. (18) has dimension (and the meaning) of the familiar vacuum gluon condensate, $\sigma^2 \sim \left\langle \alpha_s (F^a_{\mu\nu})^2 \right\rangle$, that emerged in the context of QCD sum rules (41).

Literally speaking, the ansatz Eq. (18b) cannot be correct since such a Green function would violate causality.[6] In reality, the gluon (as well as the quark) propagator should have a more sophisticated analytic structure with singularities on unphysical sheets, which would correspond, in the standard field-theoretical language, to *unstable* particles.

At the same time, the modification of the Coulomb (ghost) propagator due to restricting the functional integral to the fundamental domain (25) resulted in the singular small momentum behaviour

$$G(k) \propto \frac{1}{N_c\, g_s^2} \cdot \frac{\sigma}{\mathbf{k}^4},$$

corresponding to a linear increase of the interaction energy at large distances $R = |\mathbf{x} - \mathbf{y}|$ between colour charges, $V(R) \propto \sigma R$.

The idea of confinement emerging from dressed Coulomb exchange was further explored by Zwanziger (42) who has recently shown (43) that for the static interaction potential $V(R)$ the following inequality holds:

$$V(R) \leq V_{\mathrm{Coul}}(R). \tag{19}$$

This means that if confinement exists in pure gluodynamics, it should arise already at the dressed one-gluon exchange level — "*no confinement without Coulomb confinement*" (43). The inequality Eq. (19) appears inevitable: an inclusion of "quantum" (gluons, quarks) degrees of freedom can only *screen*, that is, suppress, the classical (Coulomb) interaction, as we have discussed in the beginning of this Section.

Recent lattice studies of the correlator of timelike link variables in Coulomb gauge (44) show that the Coulomb interaction energy of static quarks indeed

[6] It is unfortunate, therefore, that the form Eq. (18b) which Gribov suggested and discussed for illustrative purposes only, is often referred to in the literature as "the Gribov propagator".

grows linearly at large quark separations. This simulation was designed to measure the *pure* Coulomb energy, without admixture of additional "constituent" gluons,

$$\Psi \;=\; \bar{q}^{a}(\mathbf{x})\; q^{a}(\mathbf{y})\Psi_0. \tag{20}$$

The corresponding string tension σ turned out to be several times greater than the generally accepted value that originated from lattice measurements of the *full* static interaction energy.

This fact is remarkable. It shows that the Coulomb exchange excites transverse vacuum gluons inclusion of which results in the *energy gain*. These gluons "shake" the initial string Eq. (20) and *soften* it.

In this picture the typical transverse size of the string grows *logarithmically* with quark separation (the so-called "roughening"). It is worthwhile to notice that this phenomenon is similar to the increase of the size of a hadron at high energies as described by "Gribov diffusion", multiplication of the number of gluons being the physical reason for both.

3.5 Perturbative (Gluon) Confinement

Motivated by Gribov ideas, J. Greensite and C. Thorn recently suggested and developed an interesting model that aims at reproducing the main features of the Coulomb confinement essentially *perturbatively* (45).

They proposed the model in which gluons are arranged in *chains* that stretch between external charges. The Fock state wave function of such a chain looks like

$$\Psi_{chain}[A] = \bar{q}^{a_1}(\mathbf{x})\; A^{a_1 a_2}(\mathbf{x_1})\; A^{a_2 a_3}(\mathbf{x_2})\; \ldots\; A^{a_N a_{N+1}}(\mathbf{x_N})\; q^{a_{N+1}}(\mathbf{y})\,\Psi_0[A]. \tag{21}$$

The spatial distribution of the gluons is then determined by minimizing the total

energy.

Qualitative explanation of the Coulomb confinement in terms of "the gluon chain model" of Greensite and Thorn runs as follows. In 't Hooft's large–N_c limit ($N_c \to \infty$, $g_s^2 N_c$ fixed), the dominant diagrams are the planar ones in which each of the gluons interacts only with its nearest neighbors. (Note that the nearest-neighbor interaction can be purely Coulombic, i.e. perturbative.) This state can be approximately represented (modulo $1/N_c^2$ corrections) as a superposition of $(N+1)$ fundamental colour dipoles. As the distance between the quarks increases, so does the number of gluons (dipoles) N between them, so that the number of gluons per unit length turns out to stay finite, $N/R = 1/\ell =$const, as Monte Carlo simulations showed. If $E_{\rm gluon}$ is the kinetic plus nearest-neighbor interaction energy of each gluon, then the total energy of the system is

$$E(R) = N E_{\rm gluon} = \frac{E_{\rm gluon}}{\ell}\; R \equiv \sigma R, \tag{22}$$

where the the rôle of the string tension is played by gluon energy per unit length, $\sigma = E_{\rm gluon}/\ell$.

It has been argued (45) that this model of confinement explains the Casimir scaling of the string tension observed on the lattice (the proportionality of the confining force between static sources in representation r of the gauge group to the quadratic Casimir operator C_r of the representation) and yields the expected "center dependence" (the dependence of the string tension on the transformation properties of the center subgroup of the gauge group Ω — "N-ality").

Thus the linear growth of the interaction potential with distance is the consequence of the linear increase in the number of gluons excited by the static quark–antiquark interaction (45). It is important to stress that N linearly increasing with distance in the *static* problem, translates into the uniform rapidity

distribution of excited gluons in the *relativistic* situation when external sources fly away with the speed of light as, for example, in electron-positron annihilation $e^+e^- \to q\bar{q} \to$ hadrons.

Such a picture therefore directly relates the notion of a nearly constant confining inter-quark force that is used to describe mass spectra of heavy quark bound states (46), with well developed "string" phenomenology of multiparticle production in high energy interactions (47). The latter successfully describes the gross features of both soft and hard phenomena. On the *soft* side we have "multiperipheral" hadroproduction in hadron–hadron reactions (Gribov plateau = Pomeron); on the *hard* side — the famous Feynman plateau, $xD(x) \simeq$const., characteristic for parton distributions that explain spacelike deep inelastic scattering (and particle content of timelike jets) at small Bjorken (Feynman) x.

This important link is not a monopoly of the gluon chain model. Indeed, we seem to be retelling the good old Kogut–Susskind confinement story: gluon field lines forming a string, digging up quark–antiquark pairs from the vacuum, a'la Schwinger tunneling mechanism in a constant field, assembling colourless hadrons in the final state. There is a crucial difference, however. We were used to look at the Kogut–Susskind scenario as calling for essentially non-perturbative dynamics: strong fields, "superconductivity" vortex picture to explain one-dimensional nature of the "string", etc. Now we are lead to think that the same physical picture can be achieved by purely *perturbative means*, that is, by employing the language of the fundamental degrees of freedom of the theory — gluons — which gluons interact in a Coulomb-like (again, perturbative) fashion.

't Hooft (48) has advanced the Greensite–Thorn arguments, and pointed out that the "gluon chains" cannot provide a complete set of states to confine the

sources. The problem becomes apparent if we turn to the real world with light quarks present. Indeed, by pulling apart two heavy quarks we expect to find ourselves holding two colourless D-mesons, in the end of the day. Obviously, this cannot be achieved without including light vacuum quarks in the game. (Formally speaking, the adjoint representation gluons cannot fully blanch colour charges in the fundamental representation — the external quarks.) Nevertheless, since gluon chain states appear to be energetically profitable, as recent lattice results have indicated (45), this model can serve as a good starting point, as a "first approximation" to the problem of quantitatively approaching the physics of hadrons. In particular, it may help to understand puzzling *softness* of the transformation of partons into hadrons. These phenomena that one observed studying energy and angular distributions of soft particles produced in hard interactions known as "local parton–hadron duality" (49), for recent reviews see (50, 51).

't Hooft's (unpublished) remark (48) bears an elegant strikingly simple title *Perturbative Confinement*. Looking back into historical perspective, we find this rather ironic, since this very endeavour — perturbative approach to the confinement problem — motivated the NPB referee to initially reject [7] the pioneering Gribov paper (40).

Instability of the perturbative vacuum tells us that in order to have a chance to approach the true *groung state* of the theory by adiabatically switching on the interaction, one has to start from an *excited* state, in terms of non-interacting perturbative vacuum. Implementation of this idea is being developed by P. Hoyer and S. Peigne, who are trying to model non-trivial structure of the vacuum by

[7] The second reason being, that "the confinement problem had been already solved and isn't worth talking about."

explicitly adding condensate terms to perturbative quark and gluon propagators (52).

3.6 OPE and Linear Potential

Another way of approaching the problem of confinement is to start at short distances, use the operator product expansion (OPE) and to study the structure of power corrections. For the case of potential acting between massive quarks inside a colourless bound state, the expansion takes the form (53), see (54) for a recent survey,

$$V(R) \overset{R\to 0}{\simeq} -\frac{C_F\alpha_s(R)}{R}\left(1+\sum_n a_n\alpha_s^n + c_3\frac{\left\langle\alpha_s(F^a_{\mu\nu})^2\right\rangle}{\alpha_s^2}\cdot R^4\right), \tag{23}$$

where the second term in the brackets describes the higher order perturbative corrections, and the third one is the leading power correction. The latter is motivated by the OPE that predicts the leading non-perturbative power correction to the vacuum energy to scale in accordance with the dimension of the vacuum expectation value of the first gauge invariant vacuum operator, $[\left\langle\alpha_s(F^a_{\mu\nu})^2\right\rangle] = [R^{-4}]$. Within perturbation theory, its origin can be traced to "infrared renormalons", for a review see (55).

What is most remarkable about the expression Eq. (23) in the context of our discussion is that it does not contain a term linear in R and thus contradicts the expected form

$$V(R) \simeq -\frac{c}{R} + \sigma R, \tag{24}$$

borne out both by phenomenology of heavy quarkonia and by the lattice studies, as we have discussed above. We therefore come to the troubling conclusion that the OPE is inconsistent with the linear confining potential.

Yet another unexpected blow to the OPE ideology came from the direct state-of-the-art Monte Carlo simulation of the vacuum condensate in lattice gauge theory (56) which showed that the leading non-perturbative correction to the Wilson loop plaquette expectation value scales with lattice spacing as a^2, instead of the OPE-blessed a^4.

Various options were suggested to avoid this contradiction: (see (54) and references therein). One can invoke new, genuinely non-perturbative, degrees of freedom as means for constructing missing dimension two vacuum condensates. The vocabulary of such approaches includes the notions of Dirac strings, (clustering) Monopoles, (percolating) Vortices and alike (57).

Alternatively, one may look at the QCD coupling itself; in other words, to search for $1/Q^2$ corrections emerging from the dressed gluon propagator.

3.7 Infrared-Finite QCD Coupling

It is interesting that such terms naturally appear when one tries to implement the idea of *analyticity* in the Q^2 variable by assuming the existence of spectral representation for the running coupling (58,59). For example, a $1/Q^2$-suppressed correction emerges as a result of simple removal of the Landau pole from the one-loop coupling (60):

$$\alpha_s(Q^2) \implies \alpha_s(Q^2) = \frac{4\pi}{b_0}\left(\frac{1}{\ln(Q^2/\Lambda^2_{QCD})} + \frac{\Lambda^2_{QCD}}{\Lambda^2_{QCD} - Q^2}\right). \qquad (25)$$

The ansatz Eq. (25) may turn out to be too simplistic.[8] Nevertheless, the idea of enforcing analyticity (causality) on α_s turned out to be very efficient, see in particular (61). The corresponding technology that improves perturbative expansions is known under the name of "Analytic Perturbation Theory" (62). Impressive

[8]The sign of the $1/Q^2$ term it produces does not match that of the lattice result (54).

recent results of refurbishing of perturbative series can be found in (63).

A QCD couling that stays *finite* all over the complex Q^2-plane (as an analytic $\alpha_s(Q^2)$ obviously does) allows one to peek into the strong interaction domain. One can aim at quantifying genuine confinement effects in QCD observables by linking power-suppressed non-perturbative contributions with momentum integrals (moments) of the coupling in the infrared region (59).

Markedly, the average value of the coupling that emerges from the study of the leading $1/Q$ power corrections to various jet shape observables in e^+e^- annihilation and DIS turns out to be

$$a_0 \equiv \left\langle \alpha_s(Q^2) \right\rangle = \frac{1}{2\,\mathrm{GeV}} \int_0^{2\,\mathrm{GeV}} dQ\, \alpha_s(Q^2) \;=\; 0.47 \pm 0.07\,. \tag{26}$$

(For a recent review see (51) and references therein.) Let us mention, in passing, that this number turns out to be suspiciously close to the corresponding integral over Eq. (25). Even more interesting a "coincidence" is that the "measured" value Eq. (26) is comfortably above the so-called critical value of the strong coupling which is necessary, as we shall discuss in the next Section, to trigger the light quark confinement, according to Gribov, see Eq. (28) below.

The quest for defining α_s at large distances has a long and turbulent history, for reviews see (64,65,66). On one hand, it goes without saying that the "Landau pole" is an artifact. On the other hand, in QCD we don't have means for defining the true "physical coupling" unambiguously, in contrast with QED where $\alpha(0)$ is directly accessible via small angle scattering, Josephson effect and alike. On the phenomenological side, Mattingly and Stevenson (64) have assembled an impressive list of practical applications which consistently pointed at $\alpha_s/\pi = 0.2-0.3$ as a reasonable magnitude of the "long-range" QCD interaction strength. The applications they considered ranged from rather naïve estimates of hadron-hadron

cross sections and form factors to the well elaborated Godfrey–Isgur relativized QCD quarkonium model that described quite successfully particle spectroscopy from pions all the way up to the Υ family. On the theory side, this quest ascends to the notion of "effective charges" introduced by G. Grunberg back in 1984 (67). In the context of the hunt for confinement effects, the concept of an infrared-finite coupling was suggested in (68) for the purpose of quantifying non-perturbative Λ_{QCD}/M_Q terms in fragmentation functions of heavy quarks. Perturbative *ideology* behind this concept was laid down in (69).

The model coupling Eq. (25) *freezes* at a constant value ($\alpha_s(0) = 4\pi/b_0$). Though such a regime is often advocated in the literature,[9] we'd rather it *vanished* at the origin: any (even very weak) singularity at $k^2 = 0$ of the dressed gluon propagator $D(k) \propto \alpha_s(k^2)/k^2$ would introduce unwanted long-range Van-der-Waals forces between hadrons.

4 GRIBOV CONFINEMENT

Returning to the task of constructing consistent QFT dynamics of non-Abelian gauge fields, we must conclude that, in spite of many attempts, the problem of *Gribov copies* ("Gribov horizon", "Gribov uncertainties") remains essentially open today.

By the mid-80's Gribov decided, however, to change direction and not pursue *pure gluodynamics*. He did it not because of severe difficulties in describing the fundamental domain in the functional space: he always had his ways around technical obstacles. Gribov convinced himself (though not yet the physics community at large) that the solution to the confinement problem lies not in the understand-

[9] for a recent emotional review see (66)

ing of the interaction of "large gluon fields" but instead in the understanding of how the QCD dynamics can be arranged as to prevent the non-Abelian fields from growing real big. There was a deep reason for this turn, which he formulated in the following words:

> "I found I don't know how to bind massless bosons"

(read: how to dynamically construct *glueballs*).

As for fermions, there is a corresponding mechanism provided by the Fermi-Dirac statistics and the concept of the "Dirac sea". Spin-$\frac{1}{2}$ particles, even massless which are difficult to localize, can be held together simply by the fact that, if pulled apart, they would correspond to the free-fermion states that are *occupied* as belonging to the Dirac sea.

4.1 Perturbative Light Quark Confinement

As a result of the search for a possible solution of the confinement puzzle Gribov formulated for himself the key ingredients of the problem and, correspondingly, the lines to approach it:

- The question of interest is not of "a" confinement, but that of "the" confinement in the real world, namely, in the world with two very light quarks – u and d – whose Compton wave lengths are much larger than the characteristic confinement scale ($m_q \sim 5 - 10$ MeV $\ll 1$ GeV).
- No mechanism for binding massless *bosons* (gluons) seems to exist in Quantum Field Theory (QFT), while the Pauli exclusion principle may provide means for binding together massless *fermions* (light quarks).
- The problem of ultraviolet regularization may be more than a technical trick

in a QFT with apparently infrared-unstable dynamics: the ultraviolet and infrared regimes of the theory may be closely linked. Example: the pion field as a Goldsone boson emerging due to spontaneous chiral symmetry breaking (short distances) and as a quark bound state (large distances).

- The Feynman diagram technique has to be reconsidered in QCD if one goes beyond trivial perturbative correction effects. Feynman's famous $i\epsilon$ prescription was designed for (and is applicable only to) the theories with stable perturbative vacua. To understand and describe a physical process in a confining theory, it is necessary to take into consideration the response of the vacuum, which leads to essential modifications of the quark and gluon Green functions.[10]

Existence of light quarks is crucial for the Gribov confinement scenario. It is clear without going into much mathematics that the presence of light quarks is sufficient for preventing the colour forces from growing real big: light quarks in the vacuum are eager to screen any separating colour charges and turn dragged apart heavy quarks into a pair of blanched D-mesons.

The question becomes quantitative: how strong is strong? How much of a tension does one need to break the vacuum and organize such a screening?

4.2 Supercritical Binding

In a pure perturbative (non-interacting) picture, the empty fermion states have *positive energies*, while the *negative-energy* states are all filled. With account of interaction the situation may change, *provided* two *positive-energy* fermions

[10]The proper technology lies in a generalisation of the Keldysh diagram technique designed to describe kinetics out of equilibrium.

(quarks) were tempted to form a bound state with a *negative* total energy. In such a case, the true vacuum of the theory would contain *positive kinetic energy* quarks hidden inside the *negative energy* pairs, thus preventing positive-energy quarks from flying free.

A similar physical phenomenon is known in QED under the name of super-critical binding in ultra-heavy nuclei. Dirac energy levels of an electron in an external static field created by the large point-like electric charge $Z > 137$ become *complex*. This means instability. Classically, the electron "falls onto the centre". Quantum-mechanically, it also "falls", but into the Dirac sea.

In QFT the instability develops when the energy ϵ of an empty atomic electron level drops, with increase of Z, below $-m_e c^2$. An e^+e^- pair pops up from the vacuum, with the vacuum electron occupying the level: the super-critically charged ion decays into an "atom" (the ion with the smaller positive charge, $Z-1$) and a real positron

$$A_Z \Longrightarrow A_{Z-1} + e^+ \,, \qquad \text{for } Z > Z_{\text{crit.}}$$

Thus, the ion becomes unstable and gets rid of an excessive electric charge by emitting a positron (70). In the QCD context, the increase of the running quark-gluon coupling at large distances replaces the large Z of the QED problem.

Gribov generalised the problem of supercritical binding in the field of an infinitely heavy source to the case of two massless fermions interacting via Coulomb-like exchange. He found that in this case the supercritical phenomenon develops much earlier. Namely, a *pair of light fermions* interacting in a Coulomb-like manner develops supercritical behaviour if the coupling hits a definite critical value (71)

$$\frac{\alpha}{\pi} > \frac{\alpha_{\text{crit}}}{\pi} = 1 - \sqrt{\frac{2}{3}} \,. \tag{27}$$

In QCD one has to account for the colour Casimir operator. Then the value of the coupling above which restructuring of the perturbative vacuum leads to chiral symmetry breaking and, likely, to confinement (see (72) and references therein), translates into

$$\frac{\alpha_{\rm crit}}{\pi} = C_F^{-1}\left[1-\sqrt{\frac{2}{3}}\right] \simeq 0.137\,. \qquad C_F = \frac{N_c^2-1}{2N_c}. \tag{28}$$

This number, apart from being easy to memorize, has another important quality: it is numerically small. Gribov's ideas, being understood and pursued, offer an intriguing possibly to address all the diversity and complexity of the hadron world from within the field theory with a reasonably small effective interaction strength.

4.3 Gribov Equation

Gribov constructed the equation for the quark Green function as an approximation to the general corresponding Schwinger-Dyson equation. This approximation took into account the most singular (logarithmically enhanced) infrared and ultraviolet effects due to quark-gluon interactions and resulted in a non-linear differential equation which possesses a rich non-perturbative structure.

An amazing simplicity of the Gribov construction makes one wonder, why such an equation had not been discovered some 20 years earlier when a lot of effort was applied in a search for non-perturbative phenomena of the superconductivity type in QED (Nambu–Jona-Lasinio; Baker–Johnson; Fomin–Miransky et al.).

Take the first order self-energy diagram $\Sigma_1(q)$: a fermion (quark/electron) with momentum q virtually decays into a quark (electron) with momentum q' and a massless vector boson (gluon/photon) with momentum $k = q - q'$:

$$\Sigma_1(q) = [C_F]\frac{\alpha}{\pi}\int \frac{d^4q'}{4\pi^2 i}\left[\,\gamma_\mu\, G_0(q')\,\gamma_\mu\,\right]\, D_0(q-q'), \qquad D_0(k) = \frac{1}{k^2+i\epsilon}, \tag{29}$$

with G and D the fermion and boson propagators, respectively. The corresponding Feynman integral diverges linearly at $q' \to \infty$. To kill the ultraviolet divergences (both linear and logarithmic), it suffices to *differentiate* it twice over the external momentum q.

The first Gribov's observation was that $1/k^2$ of the boson propagator happens to be the Green function of the four-dimensional Laplace operator,

$$\partial_\mu^2 \frac{1}{(q-q')^2 + i\epsilon} = -4\pi^2\, i\delta(q-q'), \qquad \partial_\mu \equiv \frac{\partial}{\partial q_\mu},$$

where ∂_μ denotes the momentum differentiation. Therefore, the operation ∂_μ^2 applied to Eq. (29) takes away the internal momentum integration and leads to an algebraic expression which is *local* in the momentum space, $k = q - q' = 0$,

$$\partial_\mu^2 \Sigma_1(q) \;=\; g\,\gamma_\mu\, G_0(q)\,\gamma_\mu\,, \qquad g = \begin{cases} \dfrac{\alpha}{\pi} & \text{for QED,} \\ C_F \dfrac{\alpha_s}{\pi} & \text{for QCD.} \end{cases} \tag{30}$$

This is the "Born approximation". With account of higher order radiative corrections, the first thing that happens is that the bare fermion propagator G_0 dresses up, $G_0(q) \to G(q)$, and so do the Born vertices $\gamma_\mu \to \Gamma_\mu(q,q,0)$. The second crucial observation was that the exact vertex function $\Gamma_\mu(q, q-k, k)$ describing the emission of a *zero momentum* vector boson, $k_\mu \equiv 0$, is not an independent entity but is related with the fermion propagator by the Ward identity,

$$\Gamma_\mu(q,q,0) = -\partial_\mu G^{-1}(q)\,. \tag{31}$$

This statement is *literally* true in Abelian theory (QED), and, after some reflection, *can be made* true in the non-Abelian case (QCD) as well.

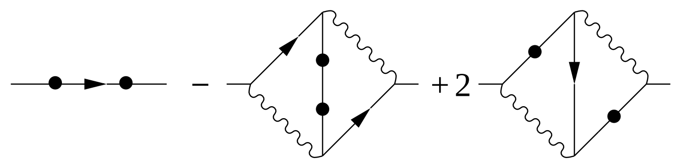
Thus, we arrived to the Gribov equation for the quark Green function (71, 72)

$$\partial_\mu^2 G^{-1}(q) = g\,\partial_\mu G^{-1}(q)\, G(q)\, \partial_\mu G^{-1}(q) + \dots\,, \tag{32}$$

where $\dots$ stand for less singular $\mathcal{O}\left(g^2\right)$ integral terms.

In principle, the whole PT series expansion may be constructed for the right hand side in terms of exact Green functions (and their momentum derivatives). In particular, with account of the first subleading terms Eq. (32) becomes an integro-differential equation and its r.h.s. may be represented in the following graphic form:

$\partial_\mu^2 G^{-1}(q) \;=$



$+\dots,$

with black dots standing for the momentum gradient of the inverse propagator – the exact zero-momentum boson emission vertex (69). Yet another set of higher order corrections makes the coupling run, $g \to g(q^2)$. In the $\left|q^2\right| \to \infty$ limit the QCD coupling vanishes due to the asymptotic freedom, and Eq. (32) turns into the free equation, $\partial_\mu^2 G^{-1} = 0$, whose general solution has the form

$$G^{-1}(q) = Z_0^{-1}\left[(m_0 - \hat{q}) + \frac{\nu_1^3}{q^2} + \frac{\nu_2^4\,\hat{q}}{q^4}\right] \qquad\qquad (\hat{q} \equiv \gamma_\mu q^\mu)\,. \tag{33}$$

This general perturbative solution has two new arbitrary parameters ν_1 and ν_2 in addition to the familiar two (bare mass m_0 and the wave function renormalization constant Z_0), since the master equation is now the *second* order differential equation, unlike in the standard renormalization group (RG) approach.

The new terms are singular at $q^2 \to 0$. Therefore in QED, for example, we simply drop them as unwanted, thus returning to the RG structure. Such a prescription, however, exploits the knowledge that nothing dramatic happens in the infrared domain, so that the real electron in the physical spectrum of the theory, whose propagation we seek to describe, is inherently that very same object that we put into the Lagrangian as a fundamental bare field.

Not so clear in an infrared unstable theory (QCD). Here we better keep all four terms in Eq. (33), wait and see.

At large virtualities, $\left|q^2\right| \gg m^2$, the two additional terms can be looked upon as power suppressed corrections that emerge due to non-trivial structure of the QCD vacuum. The corresponding dimensional parameters can be directly linked with the non-perturbative vacuum condensates,

$$\nu_1^3 \propto \left\langle \bar{\psi}\psi \right\rangle , \qquad \nu_2^4 \propto \left\langle \alpha_s F_{\mu\nu}^a F_{\mu\nu}^a \right\rangle ,$$

which constituted the core of the "ITEP sum rules" famous for successful phenomenology of low-lying hadron resonances, see (73).

Moreover, in the finite momentum region, $\left|q^2\right| \sim m^2$, where all four terms have to be treated on the same footing, Gribov found *bifurcation* – a non-perturbative solution – emerging in Eq. (32) *if* the coupling in the infrared region exceeded the critical value Eq. (28). The new phase corresponds to *spontaneously broken chiral symmetry*. This means that given a supercritical coupling in the infrared, the quark Green function may possess a non-trivial mass operator even in the chiral limit of vanishingly small bare (ultraviolet) quark mass $m_0 \to 0$ (71, 72).

4.4 Critical Coupling and Chiral Symmetry Breaking

In this section we suggest a simple algebraic exercise that should help a reader to see the origin of the critical coupling and to appreciate the Gribov equation as a tool for grasping dynamical breaking of the chiral symmetry.

Let us introduce

$$A_\nu(q) \;\equiv\; \partial_\nu G^{-1}(q) \cdot G(q). \tag{34}$$

Observing that

$$\partial_\nu^2 G^{-1} - \partial_\nu \left(\partial_\nu G^{-1} \cdot G \right) = -\partial_\nu G^{-1} \cdot \partial_\nu G = \left(\partial_\nu G^{-1} \cdot G \right) \left(\partial_\nu G^{-1} \cdot G \right) ,$$

the master equation Eq. (32) can be cast in the following compact form,

$$\partial_\nu A_\nu + (1-g)\, A_\nu A_\nu = 0. \tag{35}$$

Using the standard representation for the fermion propagator in terms of the wave function renormalization factor Z and the running mass m,

$$G^{-1}(q) = Z^{-1}(\xi)\,[\, m(\xi) - \hat{q}\,]\,, \qquad \xi = \ln q \equiv \ln \sqrt{q^2},$$

and introducing *anomalous dimensions*

$$\Gamma = \frac{\dot{Z}}{Z}, \qquad \Gamma_m = \frac{\dot{m}}{m} \qquad \left(\dot{f} \equiv \frac{df}{d\xi}\right), \tag{36}$$

it is straightforward to derive an explicit expression for the "vector field" A_ν:

$$A_\nu = \frac{1}{q}\left[\sigma_{\nu\mu}n^\mu + (1-\Gamma)\,n_\nu - \frac{m}{q}(\Gamma_m - \Gamma)\hat{n}n_\nu\right]\cdot\frac{1+\hat{n}m/q}{1-(m/q)^2}. \tag{37}$$

Here

$$n_\nu = \frac{q_\nu}{q},\;\; n^2 = 1; \quad \sigma_{\nu\mu} = \frac{1}{2}(\gamma_\nu\gamma_\mu - \gamma_\mu\gamma_\nu),\;\; \sigma_{\nu\mu}\sigma_{\mu\rho} = -3g_{\nu\rho}.$$

In the region of relatively large momenta, $q \gg m$, we have

$$q\cdot A_\nu \;=\; [\,\sigma_{\mu\nu}n^\mu + (1-\Gamma)n_\nu\,] \;+\; \frac{m}{q}\,(\gamma_\nu - \Gamma_m\hat{n}n_\nu) + \mathcal{O}\left(\frac{m^2}{q^2}\right).$$

Applying this approximation to Eq. (35) gives

$$q^2\partial_\nu A_\nu \;\simeq\; \left[2(1-\Gamma) - \dot{\Gamma}\right] - \left(\dot{\Gamma}_m + \Gamma_m^2 + 2\right)\frac{m}{q}\cdot\hat{n}; \tag{38a}$$

$$q^2 A_\nu A_\nu \;\simeq\; \left[(1-\Gamma)^2 - 3\right] \,+\, 2(1-\Gamma)(1-\Gamma_m)\frac{m}{q}\cdot\hat{n}. \tag{38b}$$

Equating the scalar and $\hat{n}$ terms in Eq. (35) produces then a system of coupled differential equations for anomalous dimensions:

$$\dot{\Gamma} \;=\; (1-g)(1-\Gamma)^2 + 2(1-\Gamma) - 3(1-g), \tag{39}$$

$$\dot{\Gamma}_m \;=\; -\left(\Gamma_m^2 + 2\right) + 2(1-g)(1-\Gamma)(1-\Gamma_m). \tag{40}$$

It is its stable point $\dot{\Gamma} = \dot{\Gamma}_m = 0$ which determines the ultraviolet anomalous dimensions Γ^* and Γ^*_m. The first equation $\dot{\Gamma} = 0$ is self-contained and results in

$$(1-g)(1-\Gamma^*) = \sqrt{3(1-g)^2+1} - 1, \qquad \Gamma^* = \frac{2-\sqrt{3(1-g)^2+1}-g}{1-g}, \quad (41)$$

where the sign of the square root was chosen such as to select among the two fixed points in Eq. (39) the *stable* one.[11] The anomalous dimension of the mass in Eq. (40) is driven by that of the wave function, since from $\dot{\Gamma}_m = 0$ we have

$$\Gamma^*_{m\pm} = -(1-g)(1-\Gamma) \pm \sqrt{[(1-g)(1-\Gamma)+1]^2 - 3}\,. \qquad (42)$$

Substituting Γ from Eq. (41), for the stable point we finally obtain

$$\Gamma^*_m = \Gamma^*_{m+} \;=\; 1 - \sqrt{3(1-g)^2+1} + \sqrt{3(1-g)^2-2}. \qquad (43)$$

Expressions Eqs. (41)–(43) are *non-perturbative* since they have emerged from *non-linear* Eq. (32). The first term of the series expansion in g,

$$\Gamma^*(g) = \frac{1}{2}\,g + \mathcal{O}\left(g^2\right), \qquad \Gamma^*_m(g) = -\frac{3}{2}\,g + \mathcal{O}\left(g^2\right),$$

coincides with the known one-loop perturbative expression for the anomalous dimension of the (Feynman gauge) fermion wave function and of the (gauge invariant) mass operator, respectively.

Now we can readily see how the *critical coupling* emerges that have been announced above in Eq. (27). Indeed, when g reaches the value $g_{\rm crit} = 1 - \sqrt{\frac{2}{3}}$, the running mass becomes *complex* due to the term $\sqrt{3(1-g)^2-2}$ in Eq. (43).

This signals instability. In fact, at this point the two anomalous dimensions of Eq. (42) become equally important, $\mathrm{Re}\,\Gamma^*_{m\pm}(g=g_{\rm crit}) = -(\sqrt{3}-1)$. The mass operator $m(\xi)$ remains real but ceases to be monotonic and starts to oscillate with

[11]The second solution corresponds to $\Gamma^*(g=0) = 4$ and describes renormalization of the power suppressed $Z^{-1}\nu_2^4/q^4$ term of the free fermion propagator Eq. (33).

$\xi = \ln q$. It is this oscillatory behaviour that allows one to construct a symmetry breaking solution corresponding to $m_0 = 0$. Its mass operator is regular at $q = 0$ and decays fast in the ultraviolet, $m(\xi) \propto \exp(-2\xi) \propto 1/q^2$, where the chiral symmetry gets dynamically restored.

For explicit construction of this chiral symmetry breaking solution, its properties and physics of accompanying Goldstone states see (72).

4.5 Quarks, Pions and Confinement

As far as *confinement* is concerned, the approximation Eq. (32) turned out to be insufficient. A numerical study of the Gribov equation carried out by C. Ewerz showed (74) that the corresponding quark Green function does not possess an analytic structure that would correspond to a confined object.

Given the dynamical chiral symmetry breaking, however, the Goldstone phenomenon takes place bringing pions to life. In his last paper (75) Gribov argued that the effects that Goldstone pions induce, in turn, on the propagation of quarks is likely to lead to confinement of light quarks and, as a result, to confinement of any colour states.

The approximate equation for the Green function of a massless quark, which accommodates a feed-back from Goldstone pions reads (75)

$$\begin{aligned} \partial_\mu^2 G^{-1}(q) &= g(q)\, \partial_\mu G^{-1}(q)\, G(q)\, \partial_\mu G^{-1}(q) \\ &\quad - \frac{3}{16\pi^2 f_\pi^2} \left\{ i\gamma_5, G^{-1}(q) \right\} G(q) \left\{ i\gamma_5, G^{-1}(q) \right\}. \end{aligned} \tag{44}$$

It is important to notice that since pions have emerged *dynamically* in the theory, their coupling to quarks is not arbitrary but is tightly linked with the quark propagator itself (search for an anti-commutator of γ_5 with G^{-1} in Eq. (44)). Moreover, the pion–axial current transition constant f_π is not arbitrary either,

but has to satisfy a definite relation which, once again, is driven by the behaviour of the exact quark Green function:

$$
\begin{aligned}
f_\pi^2 &= \frac{1}{8}\int \frac{d^4q}{(2\pi)^4 i}\,\mathrm{Tr}\left[\,\left\{i\gamma_5, G^{-1}\right\}\,G\,\left\{i\gamma_5, G^{-1}\right\}G\left(\partial_\mu G^{-1}\,G\right)^2\,\right] \\
&+ \frac{1}{64\pi^2 f_\pi^2}\int \frac{d^4q}{(2\pi)^4 i}\,\mathrm{Tr}\left[\,\left(\left\{i\gamma_5, G^{-1}\right\}\,G\right)^4\,\right]. \qquad (45)
\end{aligned}
$$

The second of the two papers (72, 75) concluding Gribov's study of the light-quark supercritical confinement theory remained unfinished. It ends abruptly in the middle of the discussion of the most intriguing question, namely, what is the meaning, and practical realization, of unitarity in a confining theory.

The modified Gribov equation Eq. (44) still awaits a detailed study aiming at the analytic structure of its solutions.

4.6 Gluon Sector

Another important open problem is to construct and to analyse an equation for the gluon similar to that for the quark Green function, from which a consistent picture of the coupling $g(q)$ rising above the critical value in the infrared momentum region should emerge.

The difficulty with the gluon sector of the theory lies in separating the running coupling effects from an unphysical gauge dependent phase that are both present in the gluon Green function. To analyse renormalization of the gluon Gribov has used in (72) the Duffin–Kemmer formalism (also known as "linear formalism") which treats the gluon potential A_μ^a and the field strength $F_{\mu\nu}^a$ as independent variables. Renormalization properties of the gauge invariant correlator $\langle FF\rangle$ gives then a direct access to the running coupling. The Duffin–Kemmer technique being plagued with artificial divergencies, this attempt did not result however in an equation for α_s in a closed form.

The solution may lie in using the "background gauge" or in exploiting the unitarity motivated "pinch" technique developed by J. Papavassiliou, N.J. Watson and others (see (76) and references therein), as in both approaches the QCD coupling is directly related with the renormalization of the gluon Green function.

5 QED AT SHORT DISTANCES

At the same time, to construct a smart equation for the running coupling poses no problem in an Abelian gauge theory where the boson propagator is gauge invariant.

Gribov has carried out this programme for QED. Here the coupling becomes supercritical at extremely short distances. Gribov's analysis aimed at resolving the long-standing problem of the "Landau pole" in the running QED coupling. The "Landau ghost" in the photon propagator at academically large momenta seems to be a formal problem so long as QED is actually a part of a broader field theoretical scheme. Nevertheless, its resolution is extremely important since it demonstrates what type of new, non-perturbative, phenomena in QFT one might expect when the strength of the coupling becomes large (dynamical symmetry breaking and appearance of Goldstone states, condensation, confinement, etc.).

Gribov did not finish the paper entitled "Quantum Electrodynamics at Short Distances". Handwritten notes he left behind, deciphered and translated into English by J. Nyiri, appeared in (77). It is important to mention that no attempt has been made by the editorial team to "debug" these notes.[12]

[12] This was done on purpose. By correcting omnipresent mistakes/misprints in obvious places the editors could have misled a potential reader into putting too much trust into not-so-obvious derivations and conclusions. As the matter stands now, it should be not *read* but rather *worked through* with a pencil and an A4-pad at hand. We strongly encourage to do so everyone willing

Here we will reproduce essential steps of the part of Gribov's analysis that directly addressed the "Moscow zero" problem. By so doing we hope to be able to ignite enough curiosity so that our readers will be tempted to look themselves into more delicate issues (such as appearance of light super-bound states, condensation, the Goldstone phenomenon) that were also considered in (77).

5.1 Equation for Photon Polarization Operator

Electron loop Feynman diagrams for the polarization operator $\Pi_{\mu\nu}(k)$ for a photon with momentum k diverges quadratically. Therefore in order to obtain a convergent integral (and thus a finite answer) we need to differentiate it *thrice* over k. At the same time, conservation of current dictates

$$\Pi_{\mu\nu}(k) = \left(k_\mu k_\nu - g_{\mu\nu}k^2\right) \cdot \Pi(\xi), \qquad \xi = \ln k, \tag{46}$$

where the factor Π diverges logarithmically in the ultraviolet; in the Born approximation,

$$\Pi_0 = e_0^2 \cdot \mathrm{const} \cdot \ln \frac{\Lambda_{\mathrm{UV}}}{k}.$$

Let us apply successively two differential momentum operations to Eq. (46):

$$\partial_\mu \equiv \frac{\partial}{\partial k_\mu}, \quad \partial_\mu\Big[\partial_\nu \Pi_{\mu\nu}(k)\Big] = \partial_\mu\Big[\,3k_\mu \Pi(\xi)\,\Big] \;=\; 3(4\Pi + \dot{\Pi}) \tag{47}$$

(recall that the dot marks ξ–derivative). To get rid of the remaining logarithmic divergence it suffices to differentiate Eq. (47) over ξ. This is equivalent to applying the operator $k_\sigma \partial_\sigma$ which produces

$$\dot{\Pi} + \frac{1}{4}\ddot{\Pi} \;=\; \frac{(k_\sigma\partial_\sigma)}{12} \cdot \partial_\mu\partial_\nu\, \Pi_{\mu\nu}(k).$$

to play a $\frac{1}{3}$–of–a–Rosetta–Stone quest.

The l.h.s. can be expressed directly through the running QED coupling. Indeed, observing that, by definition,

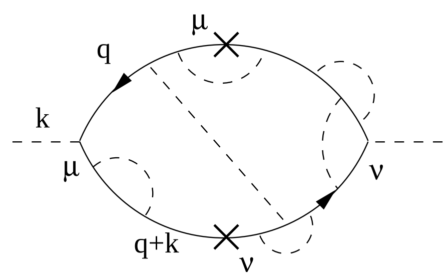
$$g(\xi) = \frac{\alpha(\xi)}{\pi} = \frac{e^2(\xi)}{4\pi^2} = \frac{e_0^2}{4\pi^2} \cdot \frac{1}{1+\Pi(\xi)},$$

we may write

$$\left(\frac{d}{d\xi} + \frac{1}{4}\frac{d^2}{d\xi^2}\right)\frac{1}{g} \;=\; \frac{(k_\sigma \partial_\sigma)}{6} \cdot \frac{2\pi^2}{e_0^2}\, \partial_\mu \partial_\nu \, \Pi_{\mu\nu}(k). \tag{48}$$

As for the r.h.s. of Eq. (48), we treat $\Pi_{\mu\nu}$ as a sum of Feynman diagrams and apply the first two differentiations *diagrammatically*. This results in

$$\frac{1}{e_0^2}\, \partial_\mu \partial_\nu \, \Pi_{\mu\nu}(k) \;=\; \sum \tag{49}$$



where crosses mark momentum derivative of the bare electron propagator,

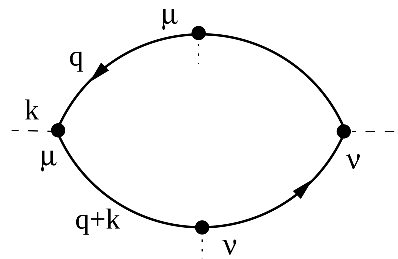
$$\partial_\mu G_0(q+k) = \partial_\mu \frac{1}{m_0 - (\hat{q} + \hat{k})} = G_0 \gamma_\mu G_0; \quad \times = -\partial_\mu G_0^{-1} = \gamma_\mu. \tag{50}$$

(Integral over the loop momentum q diverges only logarithmically which justifies integration by parts used to derive Eq. (49).) After differentiation, the overlapping divergencies disappear that plagued the original Schwinger–Dyson equation for the polarization operator. Therefore, higher order corrections combine to *renormalize* four fermion propagators, $G_0(q) \to G(q)$, both external vertices, $\gamma_\mu \to \Gamma(q, q+k, k)$, as well as the crosses:

$$\times = \gamma_\mu = -\frac{\partial}{\partial q_\mu} G_0^{-1}(q) \quad \to \quad -\frac{\partial}{\partial q_\mu} G^{-1}(q) = \Gamma_\mu(q,q,0) \;\equiv\; \bullet\,. \tag{51}$$

We thus arrive at the renormalized graph

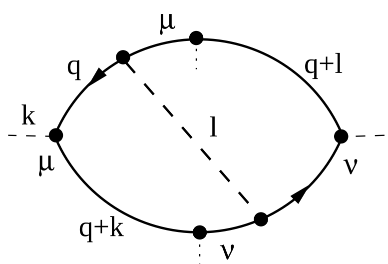
$$\frac{1}{e_0^2}\,\partial_\mu \partial_\nu\, \Pi_{\mu\nu}(k) \;=$$



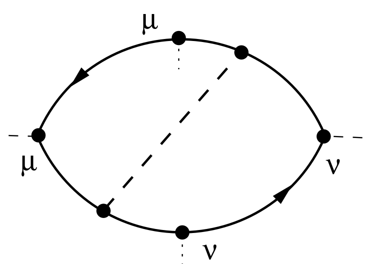
$$+\ \Delta. \tag{52}$$

For the correction term $\Delta = \mathcal{O}(g)$ a perturbative series expansion can be constructed in terms of two-particle irreducible multi-loop graphs built of exact propagators and interaction vertices,

$$\Delta \;=$$



$$+$$



$$+\,\mathcal{O}\left(g^2\right). \tag{53}$$

Feynman integral corresponding to the leading contribution in Eq. (52) reads

$$\int \frac{d^4q}{(2\pi)^4 i}\,\mathrm{Tr}\Big[\,\Gamma_\mu(q,q{+}k,k)G(q)A_\mu(q)\Gamma_\nu(q{+}k,q,-k)G(q{+}k)A_\nu(q{+}k)\,\Big], \tag{54}$$

where we have combined the adjacent dot-operator  $= \partial_\mu G^{-1}(q)$ and the Green function $G(q)$ into A_μ of Eq. (34).

We are interested solely in the logarithmically divergent contribution to Eq. (54) since finite pieces, after applying the last differentiation $(k_\sigma \partial_\sigma)$ in Eq. (48), will produce but negligible corrections that are power suppressed for large external photon momenta, $k \gg m$. Bearing this in mind, in the relevant integration region $k \ll q \ll \Lambda_{\rm UV}$ we can omit k in the loop propagators, $G(q+k) \simeq G(q)$, and in the exact photon vertices, $\Gamma_\mu(q,q{+}k,k) \simeq \Gamma_\mu(q,q,0) = -\partial_\mu G^{-1}(q)$,

In the large momentum region we invoke Eq. (38b) to obtain

$$\mathrm{Tr}\Big[\,A_\mu(q)A_\mu(q)\,A_\nu(q)A_\nu(q)\,\Big] \simeq \frac{4}{q^4}\cdot\big([1-\Gamma]^2-3\,\big)^2 .$$

Transforming the integration phase space,

$$\int \frac{d^4q}{(2\pi)^4 i\, q^4} = \frac{1}{8\pi^2}\int d\xi,$$

we derive

$$\frac{2\pi^2}{e_0^2}\partial_\mu\partial_\nu\Pi_{\mu\nu}(k) \;\simeq\; \int_{\ln k}^{\ln\Lambda_{\mathrm{UV}}} d\xi\, \frac{\mathrm{Tr}[\quad]}{4}.$$

Substituting into Eq. (48) and taking the ξ-derivative we finally arrive at

$$\left(\frac{d}{d\xi} + \frac{1}{4}\frac{d^2}{d\xi^2}\right)\frac{1}{g} \;\simeq\; -\frac{1}{6}\left([1-\Gamma]^2 - 3\right)^2. \tag{55}$$

This second order differential equation generalises the standard RG equation,

$$\frac{d}{d\xi}g^{-1} \;=\; \beta(g),$$

with the β-function expressed via anomalous dimension of the (Feynman gauge) electron wave function, $\Gamma = \Gamma^*(g)$.

5.2 Small and Large Coupling Regimes

In the small coupling regime we have $\Gamma = \mathcal{O}(g) \ll 1$ and Eq. (55) gives

$$g^{-1} \;\simeq g_0^{-1} - \frac{2}{3}\ln k,$$

which is the standard one-loop expression for the running QED coupling that formally develops "Landau pole" in the deep ultraviolet, $k \sim m_e \exp(\frac{3\pi}{2}\cdot 137)$. However, for $g = \mathcal{O}(1)$ there is no reason to neglect Γ. The latter is given by

$$\Gamma(g) \;=\; 1 - \frac{3(1-g)}{1+\sqrt{1+3(1-g)^2}}. \tag{56}$$

From this representation, which is equivalent to the original Eq. (41), we observe that Γ is monotonically increasing with g, crosses unity at $g = 1$ and tends to

$$\Gamma(g) \;\stackrel{g\to\infty}{=}\; (1+\sqrt{3}) - g^{-1} + \mathcal{O}\left(g^{-2}\right).$$

We observe that in the large coupling limit the r.h.s. of Eq. (55) becomes small,

$$\left(\frac{d}{d\xi}+\frac{1}{4}\frac{d^2}{d\xi^2}\right)\frac{1}{g} \;\simeq\; -\frac{1}{6}\cdot\frac{(2\sqrt{3})^2}{g} = -\frac{2}{g},$$

resulting in *logarithmic increase* of the coupling at large photon virtualities:

$$g(\xi) \;\simeq\; 2\cdot\xi = \ln\left|k^2\right| \gg 1. \tag{57}$$

Landau singularity has disappeared. Gribov explained this phenomenon as a compensation between the contributions to vacuum polarization from *magnetic moment* of electron $[(\sigma_{\mu\nu})^2 = -3]$ by that of its *charge* $[(1-\Gamma)^2]$ in the non-perturbative regime of large anomalous dimensions.

Numerically large coupling $g \gg 1$ does not mean that the interaction becomes really *strong*. If this were the case, the approximation based on neglecting higher order terms Δ in Eq. (52) that was adopted in Eq. (55) would have been undermined. Gribov argued that since adding a photon is accompanied by additional factors A_μ in each vertex, the relative magnitude of radiative corrections, in spite of $g \gg 1$, may turn out to be finite, $g\cdot(A_\mu)^2 = \mathcal{O}(1)$.

In fact, there is no need to analyse the structure of Δ for $g \gg 1$, which regime is actually of little interest. Indeed, as have we discussed above, already at $g = \mathcal{O}(1)$ a new phenomenon occurs, which is supercritical binding of fermion pairs. It leads to appearance of scalar/pseudoscalar "mesons" and changes drastically the behaviour of the Abelian theory in the ultraviolet.

5.3 Composite Higgs as a Super-critically Bound $\bar{t}t$ Pair

As an interesting byproduct of his supercritical quark confinement in the *infrared*, Gribov considered in 1994 an application of the supercritical picture to the weak sector of the Standard Model in the *ultraviolet*.

It is widely believed that the couplings of the basic interactions $U(1)_{\mathrm{Y}}$, $SU(2)_{\mathrm{weak}}$ and $SU(3)_{\mathrm{strong}}$ merge at some Grand Unification scale, above which the underlying dynamics becomes essentially different. However, if there is a "Super-Great Desert" instead, the Abelian hyper-charge interaction constant g_Y keeps growing with scale leading to spontaneous symmetry breaking, non-zero vacuum expectation value V, generation of W/Z masses and appearance of Higgs boson as a super-bound state of top quarks. All these phenomena being inter-connected, this scenario allows one to relate m_H and m_t with the symmetry breaking parameter $V \simeq 246\,\mathrm{GeV}$, the Weinberg angle and the values of the couplings α_s and $\alpha_{e.m.}$ at the top quark mass scale (78).

The mass of such composite Higgs particle (whose binding is similar to that of a deuteron in the zero-radius limit of nuclear forces) would have been $m_H = (170\pm7)$ GeV, given the experimental top quark mass $m_t \simeq 175\,\mathrm{GeV}$ (and varying α_s between $2m_t$ and $\frac{1}{2}m_t$).

6 CONCLUSIONS

Hadron phenomenology has accumulated a very impressive dossier of puzzles and hints, ranging from unexplained regularities in hadron spectroscopy to soft "forceless" hadroproduction in hard processes.

The reason why one keeps talking, 30 years later, about *puzzles & hints*, about *constructing* QCD rather than routinely *applying* it, lies in the conceptually new problem one faces when dealing with a non-Abelian theory with unbroken symmetry. We have to learn to master a Quantum Field Theory whose dynamics is intrinsically unstable in the infrared domain so that the objects belonging to the physical spectrum of the theory (supposedly, colourless hadrons, in the QCD con-

text) have no direct one-to-one correspondence with the fundamental fields the microscopic Lagrangian of the theory is made of (coloured quarks and gluons).

In these circumstances we don't even know how to formulate at the level of the microscopic fields the fundamental properties of the theory, such as conservation of probability (unitarity) and analyticity (causality). Indeed,

- What does **Unitarity** imply for confined objects?
- How does **Causality** restrict quark and gluon Green functions and their interaction amplitudes?
- What is the **Mass** of an INFO – [well] Identified [but] Non-Flying Object? [13]

Understanding the confinement of colour remains an open issue. The very problem can be formulated in various terms, ranging from a 10^6\$ worth a rigorous mathematical proof of the existence of a mass gap in pure gluodynamics all the way to developing down-to-earth but theory motivated practical recipes for cooking the spectrum of hadrons and predicting their production properties.

Among many a contribution to the problematics of non-Abelian gauge theories – proper quantization of Yang–Mills fields, the origin of asymptotic freedom, the nature of instantons and physics of quantum anomalies, etc., – the key ingredient of the Gribov conception of Quantum Chromodynamics was setting up the problem of the confinement of colour as that of *light quarks*.

Gribov works on gauge theories and, in particular, all his papers, talks and lectures devoted to anomalies and the QCD confinement (including the lectures

[13] The issue of quark masses is particularly damaging since a mismatch between quark and hadron thresholds (whatever the former means) significantly affects predicting the yield of heavy-flavoured hadrons in hadron collisions, markedly at LHC.

that were translated into English for the first time) were collected and recently published[14] in *Gauge Theories and Quark Confinement* (79).

From these papers, an interested reader will be able to follow the derivation of the Gribov equation and to study the properties of its (perturbative and non-perturbative) solutions, as well as to formulate and pursue the open problems awaiting analysis and resolution. Pedagogical lectures he gave in 1992 in Orsay (80) will give an opportunity to grasp the physical picture of the supercritical binding which includes an anti-intuitive notion of an "inversely populated" Dirac sea and to think about phenomenological aspects of the light quark confinement scenario.

Needless to say, the Gribov approach and recent developments reviewed here did not yet provide a consistent recipe for dealing with QCD at large distances. Nevertheless, an impressive progress has been made, and Gribov's ideas continue to inspire many of those who have contributed to it.

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[14] The book can be ordered at http://www.prospero.hu/gribov.html

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